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Aberration and the Speed of Gravity

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Abstract

Based on the observed absence of gravitational aberration, Van Flandern has recently argued that gravity has been experimentally shown to propagate at a speed of at least $2 \times 10^{10}c$. But the translation from directional observations to propagation speed is theory-dependent, and Van Flandern has made implicit assumptions about a model for gravitation that do not hold for general relativity. By explicitly computing the gravitational effect of an accelerating mass in general relativity, I show that the gravitational aberration is almost exactly canceled by velocity-dependent interactions, and that nothing need propagate faster than light. I discuss the underlying cause of this cancellation: it is required by conservation laws and by the quadrupole nature of gravitational radiation.

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In a recent paper in Physics Letters **A** [1], Van Flandern has argued that observations show that gravity propagates at a speed much greater than c. In the absence of direct measurements of propagation speed, Ref. [1] relies instead on directional information, in the form of observations of (the absence of) gravitational aberration. But the translation from a direction to a speed requires theoretical assumptions, and the implicit assumptions of Ref. [1]—in particular, that the interaction is purely central, with no velocity-dependent terms—do not hold for general relativity, or, for that matter, for Maxwell's electrodynamics.

In this paper, I explicitly compute the gravitational effect of an arbitrarily accelerating source, Kinnersley's "photon rocket" [2]. Although gravity propagates at the speed of light in general relativity, the expected aberration is almost exactly canceled by velocity-dependent terms in the interaction. While at first this cancellation seems to be "miraculous," it can be explained from first principles by turning Van Flandern's argument on its head: conservation of energy and angular momentum, together with the quadrupole nature of gravitational radiation, require that any causal theory have such a cancellation.

1 Aberration in Electromagnetism

It is certainly true, although perhaps not widely enough appreciated, that observations are incompatible with Newtonian gravity with a light-speed propagation delay added in [3,4]. If one begins with a purely central force and puts in a finite propagation speed by hand, the forces in a two-body system no longer point toward the center of mass, and the resulting tangential accelerations make orbits drastically unstable. A simple derivation is given in problem 12.4 of Ref. [4], where it is shown that Solar System orbits would shift substantially on a time scale on the order of a hundred years. By analyzing the motion of the Moon, Laplace concluded in 1805 that the speed of (Newtonian) gravity must be at least $7\times 10^6 c$ [5]. Using modern astronomical observations, Van Flandern raised this limit to $2\times 10^{10} c$ [1].

But this argument, at least in its simplest form, holds only if one postulates that the relevant force is purely central and independent of the source velocity. For Maxwell's electrodynamics and Einstein's general relativity, this assumption fails.

Let us begin with the simpler case of electromagnetism. It is well known that if a charged source moves at a constant velocity, the electric field experienced by a test particle points toward the source's "instantaneous" position rather than its retarded position. Lorentz invariance demands that this be the case, since one may just as well think of the charge as being at rest while the test particle moves. This effect does not mean that the electric field propagates instantaneously; rather, the field of a moving charge has a velocity-dependent component that cancels the effect of propagation delay to first order [6].

It is helpful to analyze this case a bit more carefully, while establishing notation that will be useful below when we consider general relativity. Let the source move along a timelike world line C in flat Minkowski spacetime, with position $z^\mu(s)$ and four-velocity $\lambda^\mu = dz^\mu/ds$. The backwards light cone from any point x^μ will intersect C at a point $z^\mu(s_R)$ (see figure 1), and this relation can be viewed as an implicit definition of the retarded proper time $s_R(x)$:

$$\eta_{\mu\nu}(x^\mu - z^\mu(s_R))(x^\nu - z^\nu(s_R)) = 0. \tag{1.1}$$

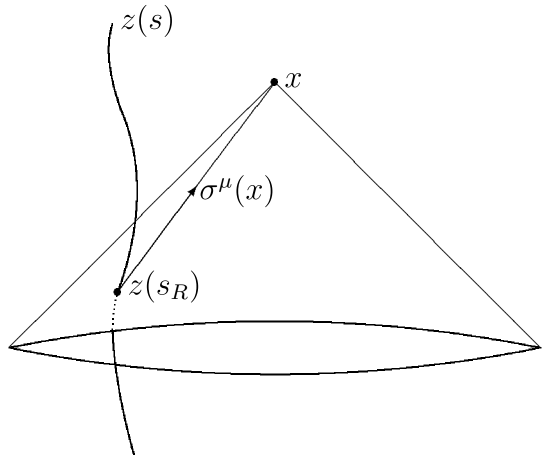

Figure 1: The geometry of retarded positions in Minkowski space

Let

$$\sigma^\mu = x^\mu - z^\mu(s_R) \tag{1.2}$$

denote the null vector connecting x and C. Differentiating (1.1), we obtain

$$\partial_\mu s_R(x) = \frac{\sigma_\mu}{r}, \tag{1.3}$$

where

$$r(x) = \lambda^\mu(s_R)\sigma_\mu \tag{1.4}$$

is an invariant retarded distance from x to C. In terms of a (3+1)-dimensional decomposition of spacetime, we have

$$\begin{aligned} \sigma^0 &= R, \quad \sigma^i = Rn^i \\ \lambda^0 &= \gamma_R, \quad \lambda^i = \gamma v_R^i, \end{aligned} \tag{1.5}$$

where $R = |\mathbf{x} - \mathbf{z}(s_R)| = t - z^0(s_R)$ is the retarded spatial distance, $\mathbf{v}_R$ is the retarded velocity, $\gamma_R = (1 - v_R^2)^{-1/2}$, and $\mathbf{n}$ is a unit spatial vector pointing toward the retarded position of the source. In "propagation-delayed Newtonian gravity," aberration appears as the fact that the force is directed along $\mathbf{n}$, and not along the vector pointing toward the "instantaneous" position of the source.

With these conventions, the Liénard-Wiechert potential in Maxwell's electrodynamics can be written as [7]

$$A^\mu = \frac{e}{r}\lambda^\mu. \tag{1.6}$$

Using standard identities [2, 8] obtained from eqn. (1.3) and the chain rule, one obtains a field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{e}{r^3}\left(1 - \sigma^\rho \frac{d\lambda_\rho}{ds}\right)(\sigma_\mu\lambda_\nu - \sigma_\nu\lambda_\mu) + \frac{e}{r^2}\left(\sigma_\mu \frac{d\lambda_\nu}{ds} - \sigma_\nu \frac{d\lambda_\mu}{ds}\right). \tag{1.7}$$

In particular, eqn. (1.5) implies that the electric field can be written as

$$E^i = F^{i0} = \frac{e}{\gamma_R^2 R^2 (1 - \mathbf{n}\cdot\mathbf{v}_R)^3}(n^i - v^i) + \textit{radiative terms}, \tag{1.8}$$

where the omitted radiative terms depend explicitly on acceleration and fall off as $1/R$ rather than $1/R^2$. Note that every term in eqns. (1.6)–(1.8) is retarded, and nothing depends on the "instantaneous" position of the source.* Nevertheless, the direction of the nonradiative "Coulomb" force in (1.8) is

$$n^i - v^i = (1 - \mathbf{n}\cdot\mathbf{v}_R)\left(n^i + (t - z^0(s_R))\frac{dn^i}{dt}\right). \tag{1.9}$$

The second term in this expression is essentially a linear extrapolation from the retarded direction n^i toward the "instantaneous" direction. In particular, for a charge in uniform motion it is easy to check that $n^i - v^i$ points toward the "instantaneous" position, so the effects of aberration are exactly canceled.

As this derivation shows, the immediate cause of this cancellation is that electromagnetism is a vector rather than a scalar interaction. Specifically, the effect of retardation in the "scalar potential" term $\partial_i A_0$ is partially compensated by the velocity-dependent "vector potential" term $\partial_0 A_i$. I will return to this point later.

Does eqn. (1.8) imply that the electric field propagates instantaneously? Clearly not. In particular, if a uniformly moving charge suddenly stops at position $z(s_0)$, the field at a distant location x will continue to point toward its "extrapolated" position—even though the charge never actually reaches that position—until the time $t - z(s_0)$ that it takes for light to travel from $z(s_0)$ to x. At that time, the field will abruptly switch direction to point toward the true position of the source. This sudden change in the field, propagating outward from $z(s_0)$ at the speed of light, is what we mean by the electromagnetic radiation of an accelerated charge. One could, of course, try to formulate an alternative model in which the Coulomb field acted instantaneously, but only at the expense of "deunifying" Maxwell's equations and breaking the connection between electric fields and electromagnetic radiation.

2 Aberration in Gravity

If we try to extend the arguments of the preceding section to general relativity, we face two subtleties. First, there is no preferred time-slicing in general relativity, and thus no unique definition of an "instantaneous" direction. For weak fields, we can use the nearly flat background to define a nearly Minkowski coordinate system, but we must expect ambiguities of order v^2. Second, we cannot simply require that a massive source accelerate. The Einstein field equations are consistent only when all gravitational sources move along the trajectories determined by their equations of motion. In particular, we can consistently represent an accelerated source only if we include the energy responsible for its acceleration.

*The potential (1.6) is, of course, gauge-dependent, and a different gauge choice such as Coulomb gauge can give it an "instantaneous" component. However, all *physical* quantities such as the electric field are gauge-invariant and are strictly retarded. See Ref. [9] for a discussion.

Fortunately, an exact solution for such an accelerated source exists. Kinnersley's "photon rocket" [2, 8, 10] represents a mass with an arbitrary acceleration brought about by the non-isotropic emission of electromagnetic radiation. Its metric, in the notation of the preceding section, is

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{2Gm(s_R)}{r^3}\sigma_\mu\sigma_\nu. \tag{2.1}$$

This metric contains four arbitrary functions of time, a time-varying mass m and the three independent components of the acceleration $d\lambda^\mu/ds$. In general, it has a nonvanishing stress-energy tensor proportional to $\sigma_\mu\sigma_\nu$, representing radiation or null dust streaming out from the world line C; it reduces to the Schwarzschild metric, with a vanishing stress-energy tensor, when m is constant and C is a straight line.

A test particle in the spacetime (2.1) will travel along a geodesic. If we use the flat metric $m = 0$ to define background Minkowski coordinates, the "acceleration" of such a particle, in Newtonian language, is determined by the connection $\Gamma^\rho_{\mu\nu}$. In particular, if the particle is initially at rest, its "acceleration" is $-\Gamma^i_{00}$. A long but routine computation yields[†]

$$\begin{aligned}\Gamma^\rho_{\mu\nu} = &-\frac{2Gm}{r^3}\eta_{\mu\nu}\sigma^\rho + \frac{Gm}{r^4}(3\lambda_\mu\sigma_\nu\sigma^\rho + 3\lambda_\nu\sigma_\mu\sigma^\rho - \sigma_\mu\sigma_\nu\lambda^\rho)\\ &- \frac{3Gm}{r^5}\left(1 - \sigma_\tau\frac{d\lambda^\tau}{ds}\right)\sigma_\mu\sigma_\nu\sigma^\rho - \frac{2G^2m^2}{r^6}\sigma_\mu\sigma_\nu\sigma^\rho - \frac{1}{r^4}G\frac{dm}{ds}\sigma_\mu\sigma_\nu\sigma^\rho,\end{aligned} \tag{2.2}$$

and in particular,

$$\begin{aligned}\Gamma^i_{00} = \frac{Gm}{R^2}\frac{1}{\gamma_R^3(1-\mathbf{n}\cdot\mathbf{v}_R)^5}\Big[&\left(1 - 2\mathbf{n}\cdot\mathbf{v}_R - 2(\mathbf{n}\cdot\mathbf{v}_R)^2 + 3v_R^2\right)n^i - (1-\mathbf{n}\cdot\mathbf{v}_R)v^i\\ &-\frac{2Gm}{R}\frac{1}{\gamma_R^3(1-\mathbf{n}\cdot\mathbf{v}_R)}n^i\Big] + \textit{radiative terms.}\end{aligned} \tag{2.3}$$

As in the electromagnetic case (1.8), the leading nonradiative term in (2.3) is proportional to $n^i - v^i$, so to lowest order there is no aberration. Now, however, there are additional corrections of higher order in v. It is not hard to show that the effect of these corrections is to further "extrapolate" from the retarded position toward the "instantaneous" position. Indeed, one finds that

$$\Gamma^i_{00} = \frac{Gm}{R^2}\frac{1}{\gamma_R^2(1-\mathbf{n}\cdot\mathbf{v}_R)^2}\left[(1+\epsilon_1)\eta^i - \frac{2Gm}{R}\frac{1}{\gamma_R^4(1-\mathbf{n}\cdot\mathbf{v}_R)^4}n^i + \epsilon_2 v^i)\right] + \textit{radiative terms} \tag{2.4}$$

where

$$\eta^i = n^i + (t - z^0(s_R))\frac{dn^i}{dt} + \frac{1}{2}(t - z^0(s_R))^2\frac{d^2n^i}{dt^2} \tag{2.5}$$

and ϵ_1 and ϵ_2 are of order v^2. In other words, the gravitational acceleration is directed toward the retarded position of the source *quadratically* extrapolated toward its "instantaneous" position, up to small nonlinear terms and corrections of higher order in velocities.

[†]Note that there is a sign error in Ref. [2].

Does eqn. (2.4) imply that gravity propagates instantaneously? As in the case of electromagnetism, it clearly does not. Every term in the connection $\Gamma^{\rho}_{\mu\nu}$ depends only on the retarded position, velocity, and acceleration of the source. The vector (2.5) does not point toward the "instantaneous" position of the source, but only toward its position extrapolated from this retarded data. In particular, as in Maxwell's theory, if a source abruptly stops moving at a point $z(s)$, a test particle at position x will continue to accelerate toward the extrapolated position of the source until the time it takes for a signal to propagate from $z(s)$ to x at light speed.

A similar result can be obtained in general relativity by systematically approximating the solution of the two-body problem [11]. As in the case considered here, the gravitational interaction propagates at the speed of light, but velocity-dependent terms in the interaction nearly cancel the effect of aberration. Indeed, it can be rigorously proven that no gravitational influence in general relativity can travel faster than the speed of light [12].

It is worth noting that the cancellation between aberration and velocity-dependent terms in general relativity is not quite exact. If gravity could be described exactly as an instantaneous, central interaction, the mechanical energy and angular momentum of a system such as a binary pulsar would be exactly conserved, and orbits could not decay. In general relativity, the gravitational radiation reaction appears as a slight mismatch between the effects of aberration and the extra noncentral terms in the equations of motion [11]. One could again try to formulate an alternative theory in which gravity propagated instantaneously, but, as in electromagnetism, only at the expense of "deunifying" the field equations and treating gravity and gravitational radiation as independent phenomena.

3 Is the Cancellation a Miracle?

We have seen that the observed lack of aberration in gravitational interactions need not imply an infinite propagation speed, but can be explained as the effect of velocity-dependent terms in the interaction. There is still something to understand, though: a cancellation as exact as that of eqns. (1.9) and (2.5) must surely have a more fundamental origin.

The answer can be obtained by standing the argument of Ref. [1] on its head. As Van Flandern emphasized, a retarded purely central force with no velocity-dependent terms inevitably leads to the drastic nonconservation of mechanical angular momentum and energy of a binary system. But by Noether's theorem, any theory derived from a Lagrangian invariant under rotations and time translations must conserve total angular momentum and energy. This is only possible if changes in mechanical angular momentum and energy are compensated by changes in the angular momentum and energy of radiation.

For electromagnetism, conservation of charge implies that there is no monopole radiation, and the power radiated in dipole radiation is proportional to $|d^2\mathbf{d}/dt^2|^2$, where $\mathbf{d}$ is the electric dipole moment of the source. Since the first derivative $d\mathbf{d}/dt$ is proportional to the velocity, a charge moving at a constant velocity can radiate no angular momentum or energy. Hence to first order in velocity, any nonconservation of mechanical angular momentum and energy due to finite propagation speed *must* be canceled by additional terms in the interaction. The exact mechanism for this cancellation may vary from theory to theory, but its existence is guaranteed by the invariance of the Lagrangian, and we can be certain that it will appear in any field equations derived from an appropriately invariant action.

For gravity, conservation of momentum and angular momentum also rule out dipole radiation [13]. The lowest order gravitational radiation is quadrupolar, and the radiated power goes as $|d^3\mathbf{Q}/dt^3|^2$, where $\mathbf{Q}$ is the mass quadrupole moment of the source. A source with a constant second derivative of $\mathbf{Q}$ can therefore radiate no angular momentum or energy, and any nonconservation of mechanical angular momentum and energy must again be canceled by additional terms in the interaction. The second derivative $d^2\mathbf{Q}/dt^2$ involves terms proportional to acceleration and to the square of the velocity, so the cancellation must occur at a higher order than it did for electromagnetism.

Note that the dipolar nature of electromagnetic radiation is intimately tied to the fact that the electromagnetic interaction is vectorial (spin 1). This explains the remark at the end of section 1: the first-order cancellation of aberration arises from the relationship among components of the vector potential, and would not appear in a scalar theory. Similarly, the quadrupolar nature of gravitational radiation is tied to the traceless tensorial (spin 2) form of the interaction. It would be interesting to trace the second-order cancellation (2.5) more concretely to this fact.

Finally, let us return to the question asked in Ref. [1]: what do experiments say about the speed of gravity? The answer, unfortunately, is that so far they say fairly little. In the absence of direct measurements of propagation speed, observations must be filtered through theory, and different theoretical assumptions lead to different deductions. In particular, while the observed absence of aberration is consistent with instantaneous propagation (with an extra interaction somehow added on to explain the gravitational radiation reaction), it is also consistent with the speed-of-light propagation predicted by general relativity.

Within the framework of general relativity, though, observations do give an answer. The Einstein field equations contain a single parameter c_g, which describes both the speed of gravitational waves and the "speed of gravity" occurring in the expression for aberration and in the velocity-dependent terms in the interaction. This parameter appears in the gravitational radiation reaction in the form c_g^{-5}, and the success of the theory in explaining the orbital decay of binary pulsars implies that $c_g = c$ at the 1% level or better [14].

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